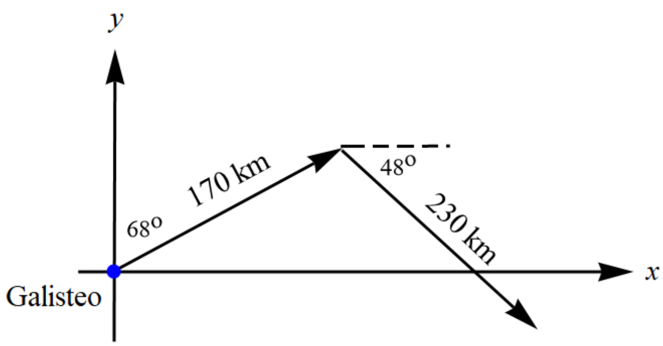
Problem 1.68

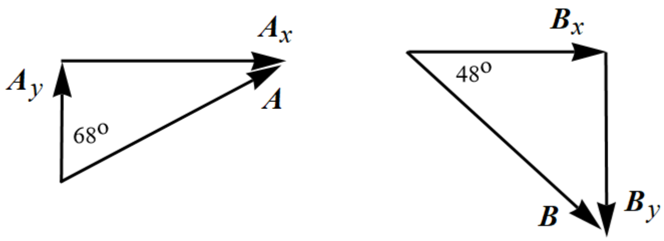
Emergency Landing. A plane leaves the airport in Galisteo and flies 170 km at 68° east of north and then changes direction to fly 230 km at 48° south of east, after which it makes an immediate emergency landing in a pasture. When the airport sends out a rescue crew, in which direction and how far should this crew fly to go directly to this plane?

Solution

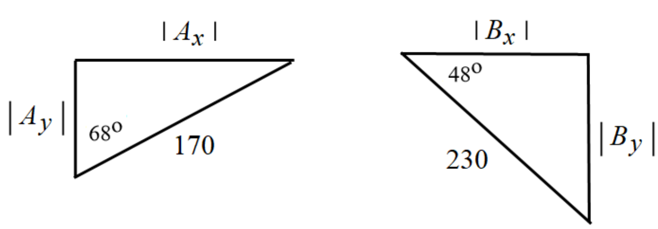
Draw the displacement vectors of the airplane.



Decompose them into components along the x- and y-axes.



Draw the triangles corresponding to the vector magnitudes.



Use trigonometry to obtain relationships involving the vector components.

$$\cos 68^\circ = \frac{|A_y|}{170} \qquad \cos 48^\circ = \frac{|B_x|}{230}$$

$$\sin 68^\circ = \frac{|A_x|}{170} \qquad \sin 48^\circ = \frac{|B_y|}{230}$$

Solve for them.

$$|A_x| = 170\sin 68^\circ$$
$$|A_y| = 170\cos 68^\circ$$

$$|B_x| = 230\cos 48^\circ$$
$$|B_y| = 230\sin 48^\circ$$

Since $\mathbf{A}_x$ and $\mathbf{A}_y$ point in the positive x- and y-directions, no minus signs are needed in the components. Since $\mathbf{B}_x$ points in the positive x-direction and $\mathbf{B}_y$ points in the negative y-direction, a minus sign is needed in the y-component but not the x-component.

$$A_x = 170\sin 68^\circ$$
$$A_y = 170\cos 68^\circ$$

$$B_x = 230\cos 48^\circ$$
$$B_y = -230\sin 48^\circ$$

The two vectors are then

$$\begin{aligned}\mathbf{A} &= \langle A_x, A_y\rangle = \langle 170\sin 68^\circ, 170\cos 68^\circ\rangle \text{ km}\\ \mathbf{B} &= \langle B_x, B_y\rangle = \langle 230\cos 48^\circ, -230\sin 48^\circ\rangle \text{ km}.\end{aligned}$$

Add these two vectors to get the position vector from Galisteo to the crash landing site.

$$\begin{aligned}\mathbf{r} &= \mathbf{A} + \mathbf{B}\\ &= \langle 170\sin 68^\circ, 170\cos 68^\circ\rangle \text{ km} + \langle 230\cos 48^\circ, -230\sin 48^\circ\rangle \text{ km}\\ &= \langle 170\sin 68^\circ + 230\cos 48^\circ, 170\cos 68^\circ - 230\sin 48^\circ\rangle \text{ km}\\ &= \langle r_x, r_y\rangle\end{aligned}$$

The magnitude of $\mathbf{r}$ gives the distance from Galisteo to the crash landing site.

$$\begin{aligned}|\mathbf{r}| &= \sqrt{(170\sin 68^\circ + 230\cos 48^\circ \text{ km})^2 + (170\cos 68^\circ - 230\sin 48^\circ \text{ km})^2}\\ &\approx 329 \text{ km}\end{aligned}$$

The counterclockwise angle θ from the positive x-axis is given by

$$\tan\theta = \frac{r_y}{r_x} = \frac{170\cos 68^\circ - 230\sin 48^\circ}{170\sin 68^\circ + 230\cos 48^\circ}.$$

Therefore,

$$\theta = \tan^{-1}\left(\frac{170\cos 68^\circ - 230\sin 48^\circ}{170\sin 68^\circ + 230\cos 48^\circ}\right) \approx -19^\circ,$$

which means the crash landing site is about 19° south of east.